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Prediction of statistical strengths of twisted fibre structures

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A new approach, based on Daniels' statistical model for a parallel fibre bundle and the recent results on yarn mechanics by the present author, is introduced to predict the strengths of twisted yarns including the effects of interfibre friction and the fibre fragmentation mechanism during yarn extension. By calculating the lateral pressure in a twisted yarn, the critical fibre length of the fibre fragmentation process has been determined, and was found to decrease together with the increasing yarn strain. This, according to the Weakest link theorem, will lead to a substantial increase of fibre strength. Incorporating this conclusion into Daniels' result yielded a more realistic prediction of the strength and its boundary for a twisted fibre structure. The key differences between a filament yarn and a short-fibre yarn and their effects on strengths are also discussed. The major results from the study are illustrated schematically.

Nomenclature

θ	The helix angle of a fibre in a yarn
q	Fibre helix angle at the yarn surface
g	The lateral pressure acting on a fibre
ε_f, $\langle \bar{\varepsilon}_f \rangle$ and ε_y	The fibre strain, the mean fibre strain and the yarn strain
σ_f and σ_{fb}	The fibre tensile stress and breaking stress
$\langle \sigma_f \rangle$, $\langle \sigma_p \rangle$ and $\langle \sigma_y \rangle$	The expected fibre strength, the strength of a parallel fibre bundle and the strength of a yarn
Θ_f, Θ_p and Θ_y	The standard deviations of the fibre strength, the parallel fibre bundle strength and the yarn strength
μ	The interfibre frictional coefficient
l_c and l_e	The critical fibre length and the effective fibre length
V_f	The fibre-volume fraction (the specific volume) of the yarn
n	The cohesion factor reflecting the fibre gripping effectiveness of the yarn
G_{TL}	The longitudinal shear modulus of the yarn
ν_{LT}	The yarn Poisson's ratio governing induced transverse strains of the yarn due to axial extension.

1. Introduction

In industrial practice, there exist two categories of twisted fibre structures in terms of fibre length used, i.e. the continuous long-fibre (filament) system and the short-fibre (staple) system. For brevity, these structures will often be referred as "yarns" in this article. Study of the yarn strength has been a topic of many research reports [1–4].

The role of twist in continuous filament yarn is mainly to produce a coherent structure that cannot readily be disintegrated by lateral actions. Twist, therefore, is not essential in offering tensile strength to the structure, and it will, in fact, lower the strength of the yarn because of the induction of fibre obliquity [1]. However, yarn twist in short-fibre yarns has the primary function of causing the fibres to be bound together by friction to form a strong yarn. Twist is hence fundamental to provide a certain minimum coherence between fibres, without which a staple fibre yarn having a significant tensile strength cannot be made. This coherence is dependent on the frictional forces brought into play by the lateral pressures between fibres arising from the application of a tensile stress along the yarn axis. Nevertheless, it has been widely accepted that when twist level is high enough, the effect of the fibre ends will become negligible. As a result, both filament and staple yarns can be treated as being equivalent. Because of this equivalence, the approach proposed in this article should be applicable to both cases when the twist level is high.

It is well known that, because of the variations in fibre strength, the breaking stress of a parallel fibre bundle deviates from that of its constituent fibres. On the other hand, prediction of strength of a twisted fibre structure is also different from that of a parallel fibre bundle because in the latter case, the effect of fibre interaction is negligible, and also as fibres are all parallel to the axis of the assembly (the loading direction) in the parallel bundle, the fibre obliquity factor is excluded. Moreover, strength prediction of a fibre structure is unlike its modulus prediction, because the strength of a material is not a volume-average quantity but rather an extremum quantity, dictated by the weakest cross-section of the structure. This so-called weakest link theorem was first elucidated by Peirce

[5] in 1926 and has since been thoroughly discussed by numerous authors.

Daniels [6] demonstrated through some rather cumbersome analysis that the asymptotic strength distribution of a parallel fibre bundle when the bundle size, N, is large enough, is of normal type. This conclusion has been accepted by the latter studies. Harlow and Phoenix [7] proposed the concept of the chain-of-bundles model of the strength of fibrous structure to tackle the issue of statistical nature of strength of individual filament, the size (length) effect on filament strength, as well as the load-sharing mechanism during structure breakage. Phoenix [8] also extended their method to the analysis of twisted fibre bundles by incorporating the fibre helical paths into his model. However, exclusion of the effects of fibre interaction, such as interfibre friction and lateral constraint in his model, brings serious limits to his theory in terms of the applicability and the accuracy of prediction.

In order to better understand and more accurately predict the strength of twisted fibre structures, besides the properties of the constituent fibres, the interactions between fibres as well as between fibre and the yarn structure have to be considered because these interactions will significantly alter the *in situ* fibre properties. Research on the prediction of yarn strength hence involves the investigation of the flaw distribution along the fibre length and its effect of yarn strength, the fibre *in situ* mechanical properties, and the stress transfer from the broken fibre into still surviving fibres during yarn extension. Some of these issues are also of interest for researchers in fibre composite materials, and some are fundamental to materials science.

Besides the weakest link mechanism, there is another mechanism known as the fragmentation process which relates to but acts differently from the weakest link mechanism. It was observed during the fracture process of both composites [9] and yarns [10] that the constituent fibres break repeatedly with increasing strain of the structure before overall material failure. This phenomenon indicates a fact that, contrary to common assumption, a broken fibre can again build up tension, carry load, break into even shorter segments and contribute towards the overall system strength.

On the other hand, because of the length–strength dependency implied by the weakest link theory, the strengths of these fibre segments will become higher with the decreasing length. It was reported by the present author and others [11] that even excluding the fibre obliquity effect, which will lower the contribution of individual fibres towards the yarn strength, the experimentally determined yarn breaking load is still greater than the prediction based on the pre-determined breaking force of all its fibres. We, therefore, proposed a new mechanism to account for this discrepancy. Given the fact that the shorter fibre will show a higher breaking strength, we suggested that because of the fragmentation process, the twist-induced lateral pressure substantially increases the overall strength of a filament yarn by increasing the apparent strengths of the segments of each individual fibre. This also suggests the assistance of a structure to its constituent components during loading.

However, there is a difference in the fragmentation process between fibre composites and yarns. For a fibre composite, because the bond adherence between fibres and the matrix remains largely unchanged as tension on the composite is increasing, the critical fibre length, l_c, which determines the final length of the fibre fragments can be treated as constant. However, in the case of twisted yarns, as the lateral pressure in the yarn which provides lateral constraint on fibres is dependent on the external tensile loading, the length of the fibre fragments hence changes (decreases) along with the increasing lateral pressure during yarn extension up to the ultimate yarn failure. This mechanism will vary for different yarn structures and will also be strongly influenced by the geometric, mechanical and surface properties of fibres (includng their variations) [11]. All of this considerably complicates the analysis of yarn strength.

It should be pointed out that the significance of the fibre fragmentation effect is dependent on the differences between the breaking strains of the structural components. Although this effect is more noticeable in fibrous structures where the breaking strains of the structure components (fibre and matrix material in composites or distinct fibre types in blended yarns) are remarkably different, it should also play a role in other fibre structures as long as there is a disperson in fibre breaking strain.

By incorporating the fibre fragmentation mechanism into analysis, the present author and others [11] have developed a computer stochastic model to simulate the actual fracture process of a blended filament yarn. The yarn was treated as a chain of fibre bundles; the length and hence the breaking stress of the bundle changes constantly during yarn extension. The yarn strength is predicted numerically by calculating the strength of its weakest cross-section. The present article proposes a statistical theory based on Daniels' model [6] for a parallel fibre bundle and the results on yarn mechanics by the present author [12] to study theoretically the relationships between the properties of the constituent fibres, the structural parameters of a twisted fibre structure, and its statistical strength.

There are certain assumptions adopted in this study to simplify the analysis.

1. The fibre helix angles in the yarn are uniformly distributed from zero up to the value at the yarn surface, and the fibre migration effect (change of the radial position of a fibre in the yarn) is negligible.
2. Fibre strength distribution is of the Weibull form.
3. When a fibre breaks, the load it was carrying is equally shared among the surviving fibres. The effects of stress concentration and dynamic wave propagation are ignored.

2. The critical fibre length l_c, in the fragmentation phenomenon

It is known [11] that the *in situ* properties of the constituent fibres in a yarn structure deviate from those tested before the constituents are incorporated into the structure. The yarn and its tensile loading situation will affect the fibre mechanical behaviour in two ways. One is due to the fact that fibres within the yarn are under bilateral loading conditons of axial tension and lateral compression. This will lead, as expected, to a different fibre behaviour from a uniaxial case. The second is due to the fragmentation phenomenon mentioned above, that under the constraint of lateral compression, a fibre behaves as a chain of mechanically independent segments, each of which possesses different mechanical properties both from those of other segments of the same fibre owing to the non-uniformity of the fibre, and from those of the whole fibre due to the size effect. In the present model, only the second factor is included and the deformation of fibres due to lateral pressure is ignored.

As revealed in the fragmentation process, during the yarn extension process, the fibre breakage will continue, until the length of the fibre fragments reaches a minimum value where load can no longer build up to its broken strength. This length is well known as the critical length. If σ_{fb} is the tensile stress which causes the fibre to break, it follows that this critical length, l_c is given by Kelly and MacMillan [13] as

$$l_c = \frac{r_f \sigma_{fb}}{\mu g} \quad (1)$$

where r_f is the fibre radius, μ the frictional coefficient between fibres, and g the local lateral pressure.

It will be shown later that the lateral pressure, g, on fibres is increasing during the yarn extension process until yarn failure. Thus the critical fibre length defined above will continuously decrease. Also, the lateral pressure on a fibre is dependent on the relative position of the fibre in the yarn, and will be different for fibres with different orientation angles, θ. As a result, we will have various critical length values along a yarn cross-section unless we use their statistical mean value.

3. The lateral pressure in a yarn

The lateral pressure is a key parameter in calculating the critical fibre length. However, as shown by Hearle [1], because of the differences of the fibre helical paths in the yarn, this pressure is not uniformly distributed along the cross-section of the yarn.

Although there have been a few expressions [1, 14] governing the distribution of the lateral pressure within a yarn, the present author proposed a new and simpler result [12] based on the shear lag theory first introduced by Cox [15]. Let us assume that the fibres in the yarn are all identical of radius r_f and length l_f. The shear stress, τ_f, acting at point x away from the centre of a fibre whose strain due to yarn tension is ε_f is

$$\tau_f = g_f \mu = \frac{n}{2} E_f \varepsilon_f \frac{\sinh(nx/r_f)}{\cosh(ns)} \quad (2)$$

where g_f is the lateral pressure on the fibre, $s = l_f/2r_f$ is the so-called fibre aspect ratio. The factor, n

$$n = \left(\frac{G_{TL}}{E_f} \frac{2}{\ln 2}\right)^{1/2} \quad (3)$$

is an indicator of the gripping effect of the yarn structure on each individual fibre. It was therefore named the yarn cohesion factor and is related as shown in Equation 3 to the ratio of yarn longitudinal shear modulus, G_{TL}, and the fibre tensile modulus, E_f, as well as the fibre arrangement within the yarn reflected by the analysis in [12].

It is seen from Equation 2 that the shear stress on fibres is not a constant. It will change along the radial position in the yarn with the fibre strain, ε_f, which is a function of the radial position of the fibre in the yarn, and also along the fibre length. As illustrated in [12], the shear stress possesses the maximum value at fibre ends, decreases towards the fibre centre and reaches zero at the fibre centre. However, because the fibre migration effect is excluded so that the fibre radial position in the yarn remains unchanged, it is very unlikely the lateral pressure will be different along the fibre length, and will become zero at the fibre centre. Considering the fact that the tendency of relative movement between fibres varies at different fibre portions along its length which governs the value of the frictional coefficient between fibres, we can hence assume that the frictional coefficient, μ, is not a constant and changes accordingly along the fibre length to make the lateral pressure constant. This constant lateral pressure should be equal to the maximum value of g_f occurring at the fibre ends, i.e. $x = l_f/2$

$$g = g_{f\max} = \frac{n}{2\mu} E_f \varepsilon_f \tanh(ns) \quad (4)$$

For a long-fibre yarn or a continuous filament yarn whose twist level is reasonably high so that $ns \gg 1$, there is $\tanh(ns) \to 1$. We then have

$$g = \frac{n}{2\mu} E_f \varepsilon_f \quad (5)$$

4. Relationship between fibre strain, ε_f, and yarn strain, ε_y

It shows in Equation 5 that the lateral pressure is related to fibre strain, ε_f, which is caused by the yarn strain, ε_y. For a fibre with orientation angle, θ, respective to yarn axis, if we include the Poisson's effect, we have according to Hearle [1] the relationship

$$\varepsilon_f = \varepsilon_y(\cos^2\theta - \nu_{LT}\sin^2\theta) \quad (6)$$

where ν_{LT} is the yarn Poisson's ratio governing induced transverse strains of the yarn due to axial extension. It is apparent that because of the dispersion in orientation of fibres in the yarn, a given yarn strain will lead to different fibre strains and eventually the different lengths of the critical fibre length, l_c. To simplify the theoretical analysis, we need to use the mean fibre strain. This mean fibre strain has been calculated and provided elsewhere by the present author [16] based on the assumption of a uniform distribution of θ value

$$\langle \varepsilon_f \rangle = \frac{\varepsilon_y}{4q}[2q(1 - \nu_{LT}) + (1 + \nu_{LT})\sin 2q] = \varepsilon_y \eta_q \quad (7)$$

where

$$\eta_q = \frac{2q(1 - \nu_{LT}) + (1 + \nu_{LT})\sin 2q}{4q} \quad (8)$$

is called the orientation efficiency factor, and q is the helix angle at the yarn surface and can be taken as a constant for a given yarn structre. It can be easily proved that when $q \to 0$, $\eta_q = 1$. The minimum value of $\eta_q = (1 - \nu_{LT})/2$ is achieved when $q \to \pi/2$. Thus the mean or the expected lateral pressure on fibres in the yarn can be written as

$$g = \frac{n}{2\mu} E_f \varepsilon_y \eta_q \quad (9)$$

5. Fibre strength versus fibre length

As revealed by the fragmentation phenomenon, the presence of multiple breaks along a single fibre illustrates the invalidity of the common perception that a broken fibre in a yarn ceases to contribute to yarn strength. The fact that one break at a position of a fibre does not prevent other parts on the same fibre length from being tensioned and successively broken, again implies the independence of each fibre segment in terms of mechanical response to external load owing to the constraint from the lateral pressure, although these fibre segments are physically connected to each other. In other words, the fibre is stretched segment by segment because of the lateral pressure. The length of each fibre segment is determined according to Equation 1 by the lateral pressure, the interfibre frictional property, as well as the fibre breaking strength.

On the other hand, Colemen [17], using the weakest link theory [5] and certain other general hypotheses, showed that the cumulative probability distribution function of fibres is of the Weibull type [18]. That is, for a given length, l_f, the probability of the fibre strength being σ is

$$F(\sigma) = 1 - \exp(-l_f \alpha \sigma^{\beta}) \quad (10)$$

where α is the scale parameter and β the shape parameter, and both of them are independent of fibre length, l_f. It shows clearly that when the fibre length decreases, the statistical strength of the fibre will increase. The mean or the expected value of the fibre strength $\langle \sigma_f \rangle$ is

$$\langle \sigma_f \rangle = (l_f \alpha)^{-1/\beta} \Gamma\left(1 + \frac{1}{\beta}\right) \quad (11)$$

and the standard deviation

$$\Theta_f = \langle \sigma_f \rangle \left\{ \frac{\Gamma[1 + (2/\beta)]}{\Gamma^2[1 + (1/\beta)]} - 1 \right\}^{1/2} \quad (12)$$

The fibre shape parameter, β, represents the dispersion of the fibre strength, and therefore is a more interesting variable. A greater β value indicates a small fibre strength variation or fibres more mechanically uniform. When $\beta \to \infty$, there would be no variation and fibre strength would also become independent of its length.

As stated above, during the yarn extension process, a fibre works as a chain of segments with each segment being tensioned to break. Because the length of the segment, as defined by the critical length, l_c, in Equation 1, decreases along with the increasing lateral pressure, g, the *in situ* strength of the fibre segments, σ_{fb}, will increase according to Equation 11.

Note that in Equation 1, the "current" fibre strength, σ_{fb}, updated according to the new length of the fibre segment, should be used. In addition, although both lateral pressure, g, and σ_{fb} are increasing during yarn extension, g increases much faster and in Equation 1 the net effect still leads to a decreasing value for the critical length, l_c.

6. The statistical strength of a parallel fibre bundle

Let us consider a bundle made of a very large number, N, of parallel fibres of Weibull type with equal length, l_f. Because it is a parallel bundle without any twist, we can expect all fibres to have the same strain $\varepsilon_f = \varepsilon_y$ where ε_y is the yarn (the bundle) strain. It is evident that if all the fibres were of the same strength, the strength of the bundle would be equal to that of its constituent fibres. However, because in reality there is a certain dispersion in the strength of the fibres, this bundle strength will hence obey a statistical distribution as well. This problem was first tackled by Daniels [6]. In his analysis, it is assumed that when a fibre breaks, the load it was carrying is instantaneously sheared equally among all the surviving fibres. Thus neither stress concentration nor dynamic wave propagation effects are considered. Based on Daniels' analysis, the density distribution function of the bundle strength approaches a normal form

$$h(\sigma) = \frac{1}{(2\pi)^{1/2}\Theta_p} \exp\left[-\frac{(\sigma - \langle \sigma_p \rangle)^2}{2\Theta_p^2} \right] \quad (13)$$

where Θ_p is the standard deviation of the strength

$$\Theta_p^2 = (l_f \alpha \beta)^{-2/\beta} \left[\exp\left(-\frac{1}{\beta}\right) \right] \times \left[1 - \exp\left(-\frac{1}{\beta}\right) \right] N^{-1} \quad (14)$$

and $\langle \sigma_p \rangle$ is the expected value of the bundle strength

$$\langle \sigma_p \rangle = (l_f \alpha \beta)^{-1/\beta} \exp\left(-\frac{1}{\beta}\right) \quad (15)$$

For a normal distribution, $\langle \sigma_p \rangle$ will be the maximum likelihood estimate of the bundle strength. It can be seen by comparing Equations 11 and 15 that because of fibre strength dispersion, the bundle strength $\langle \sigma_p \rangle$ is lower than the fibre strength $\langle \sigma_f \rangle$. The difference between the two will diminish when the shape parameter $\beta \to \infty$.

7. The statistical strength of a twisted filament yarn

Because the strength of a yarn equals the strength of its weakest cross-section, if we take into account the structural irregularities which exist especially in short fibre yarns and are caused mainly by the non-uniformity of fibre numbers at yarn cross-sections, the weakest yarn cross-section will be the one with minimum fibre number, N. Then the yarn can be treated as a chain of fibre bundles of length l_c as defined in Equation 1. The expected strength of the yarn $\langle \sigma_y \rangle$ is obtained by replacing the fibre length, l_f, with the critical length, l_c, into Equation 15. Furthermore, for a twisted real yarn, the fibre orientation and the fibre-volume fraction, V_f, have to be considered. Using the orientation efficiency factor, η_q, for the same reason in deriving the relationship, Equation 7, of fibre strain and yarn strain, the expected strength $\langle \sigma_y \rangle$ for a yarn can hence be expressed as

$$\langle \sigma_y \rangle = \eta_q V_f (l_c \alpha \beta)^{-1/\beta} \exp\left(-\frac{1}{\beta}\right) \quad (16)$$

and is discounted by a factor V_f due to the yarn density effect. It is readily proved according to the statistics theory that the standard deviation for twisted yarn, Θ_y, is related to Θ_p of the parallel bundle case as

$$\Theta_y = V_f \eta_q \Theta_p \quad (17)$$

When the yarn surface helix angle $q = 0$ so that $\eta_q = 1$, and the fibre-volume fraction $V_f = 1$, we have a parallel bundle case and this value, Θ_y will reduce to Θ_p.

Because of the normality of the yarn strength distribution, there is over 99% chance that the actual yarn strength will fall into the range of $\langle \sigma_y \rangle \pm 3\Theta_y$, i.e.

$$\sigma_y = \langle \sigma_y \rangle \pm 3\Theta_y \quad (18)$$

8. The statistical strength of a twisted short-fibre yarn

For strength prediction purpose, the only important difference between a filament yarn and a short-fibre yarn is in fibre length. As mentioned earlier, at high twist level, the effect of fibre length will become negligible so that a short-fibre yarn can be treated as a continuous filament yarn. Here we would like to further elucidate this issue. The effect of fibre length is reflected by the fibre ends effect, that for short-fibre yarn the yarn stress is transmitted to fibres through a frictional mechanism, and it takes a certain length for fibre tension to climb to the level determined by the yarn tension. There will be fibre slippage taking place along this length during yarn extension. This length is called the load transfer length [19] or effective length [1], and is given as

$$l_e = \frac{r_f \sigma_f}{\mu g} \quad (19)$$

Although this equation seems similar to the definition of the critical length, they are entirely different concepts. The ratio σ_f/g was given by Pan [12]

$$\frac{\sigma_f}{g} = \frac{2\mu}{n} \tanh(ns/2) \quad (20)$$

This equation becomes

$$l_e = \frac{2r_f}{n} \tanh(ns/2) \quad (21)$$

According to the result given elsewhere [12] for fibre aspect ratio $s = 500$, when yarn twist factor is reasonably high so that $q = \pi/3$, there is $n = 0.54$. We then have $l_e = 3.7 r_f$. In other words, l_e approaches the same order of the fibre radius, r_f, when the yarn twist is high enough. This proves that the fibre length effect can indeed be neglected at high twist level so that short-fibre yarn becomes equivalent to continuous filament yarn.

9. Calculation and discussion

The fibre properties used for the calculation are provided in Table I which also includes the assumed shape and scale parameters of the Weibull distribution for the fibres.

Using the fibre scale and shape parameters, the expected fibre strength can be calculated from Equation 11 as $\langle \sigma_f \rangle = 0.5$ GPa.

The yarn shear modulus has been derived by the present author [12], which has been found to be a function of fibre properties, yarn twist level (q value) and the yarn fibre-volume fraction. For simplicity, instead of using its expression, we apply the ratio of the moduli G_{TL}/E_f to our calculation. From [12], it is shown for $q = \pi/3$, $G_{TL}/E_f = 0.1$.

Next we need to determine the length of the fibre segment to be used in Equation 11 for determination of the fibre segment strength. Any fibre fragment with length longer than l_c is still able to break somewhere along its centre section as its stress exceeds its current strength, σ_{fb}. So the next lengths of the fragments actually vary in the range of $l_c/2$ to l_c, with the mean length being $3l_c/4$. Therefore, the mean length before fibres break into l_c will be $4l_c/3$. This will be the length by which the value of σ_{fb} for the new fibre segment is to be determined.

It was also demonstrated [12] that the yarn Poisson's ratio, ν_{LT}, is dependent on the yarn surface helix

TABLE I The fibre properties used for the calculation

Item	Typical value	Unit
Fibre radius, r_f	3×10^{-2}	mm
Fibre length, l_f	30	mm
Fibre modulus, E_f	5	GPa
Fibre frictional coefficient, μ	0.3	
Fibre number in yarn, N	200	
Fibre shape parameter, β	10	
Fibre scale parameter, α	20	$mm^{-1} GPa^{-1}$
Fibre aspect ratio $s = l_f/2r_f$	500 (short fibre)	
Yarn helix angle, q	$\pi/3$	
Yarn cohesion factor, n	0.54	
Yarn fibre volume fraction, V_f	0.8	

angle, q. For $q = \pi/3$, $\nu_{LT} = 0.9$. Hence the orientation efficiency factor $\eta_q = 0.4428$ for the following calculations.

First of all, using the values of the Weibull parameters in Table I and Equation 11, we can plot Fig. 1 to show the relationship between the fibre length and its (expected) tensile strength for fibres with different β levels. It is seen from the figure that when fibre length approaches zero, fibre strength will increase infinitely, whereas for a given length, the fibre with a greater β value will possess a higher strength. In other words, improvement of the fibre uniformity will yield strong fibres.

Fig. 2 shows, based on Equation 9, the connection between the mean lateral pressure, g, and the yarn strain, ε_y, at three G_{TL}/E_f ratio levels which, in fact, represent the three yarn twist levels because of the dependency of G_{TL}/E_f ratio on yarn twist. For convenience, the axis for pressure in the figure represents a relative scale of ratio g/E_f. As yarn strain increases, it increases the lateral pressure. The G_{TL}/E_f ratio has the similar effect.

The relationship between the yarn strain and the critical fibre length, l_c, is illustrated in Fig. 3 by combining the results in Equations 1 and 9 at three different β levels. The breaking stress of the fibre segment, σ_{fb}, is calculated from Equation 11 by replacing the fibre length, l_f, with the fibre segment length $4l_c/3$. As yarn strain increases, the value of l_c decreases rapidly in the begining and then approaches a more stable and very small value. The strength of the fibre segment will hence increase to a much higher level, as indicated in Fig. 1. Also, a higher β level leads to a larger l_c value, meaning a more uniform fibre will break into fewer fragments eventually.

The ratio between the expected strengths of the yarn and its constituent fibres can be derived from Equations 11 and 16 as

$$\frac{\langle \sigma_y \rangle}{\langle \sigma_f \rangle} = \left(\frac{l_f}{l_c}\right)^{1/\beta} \frac{V_f \eta_q}{\beta^{1/\beta} \exp(1/\beta)\Gamma[1 + (1/\beta)]} \tag{22}$$

This ratio is plotted against the length ratio of l_f/l_c at three β levels and shown in Fig. 4. (Note that in Fig. 4, the vertical axis is located at position $l_f/l_c = 1$ instead of the origin on the horizontal axis.) For a given fibre strength, when the effect of lateral pressure is neg-

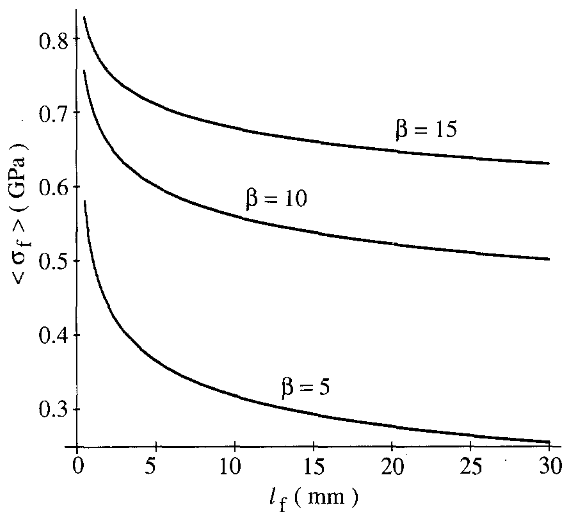

Figure 1 The fibre length, l_f, versus its expected strength $\langle \sigma_f \rangle$.

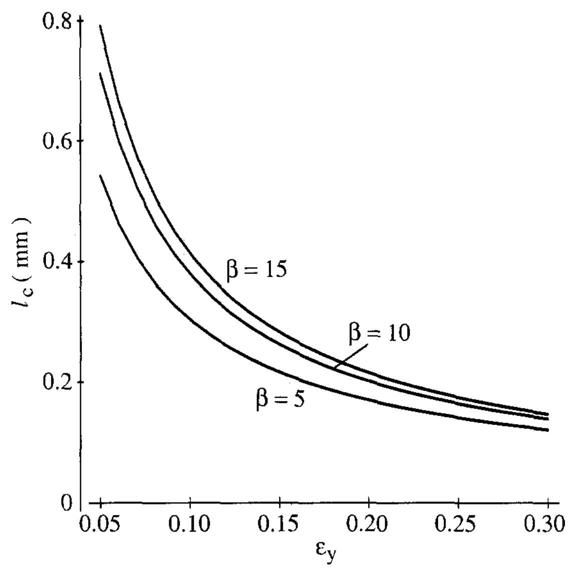

Figure 3 The critical fibre length, l_c, and the yarn strain, ε_y.

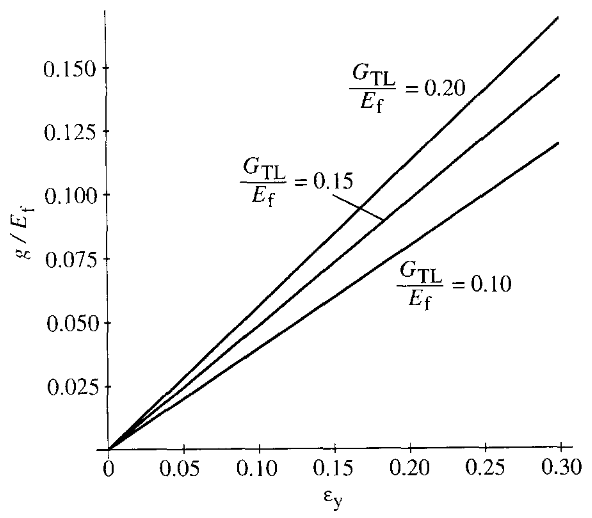

Figure 2 Dependency of g/E_f on the yarn strain, ε_y.

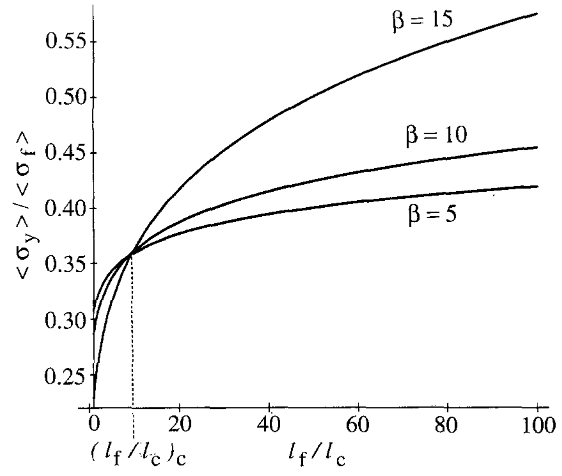

Figure 4 $\langle \sigma_y \rangle / \langle \sigma_f \rangle$ versus l_f/l_c at three β levels.

lected, we have the length ratio $l_f/l_c = 1$. The yarn strength is as low as around one-fifth of the fibre strength, depending on the β value. When the effect of the lateral pressure is considered which makes the length ratio $l_f/l_c > 1$, the yarn strength will increase as much as twice. It is interesting to see that when the yarn strain, which changes the lateral pressure and eventually the l_f/l_c ratio, increases to break the yarn, it simultaneously reinforces it. It is also interesting to note the reverse effect of the β value on the yarn strength taking place at a critical ratio $(l_f/l_c)_c$ (around 10 as depicted in Fig. 4).

Fig. 5 gives the prediction from Equation 18 of the expected yarn strength and its $\pm 3\Theta_y$ boundary as well as their relations with β. It is shown that increase of β value will increase both $\langle \sigma_y \rangle$ and, slightly, its standard deviation Θ_y. The latter is unexpected because one would assume a yarn made of more uniform fibres (with greater β value) would have a small strength variation. This will be further discussed in Fig. 6.

The variations of the yarn strength defined by the $\pm 3\Theta_y$ boundary in the figure seem much smaller than expected. This is because we have ignored the variations of yarn cross-sectional area and the yarn twist (reflected in fibre volume fraction, V_f, and the surface helix angle, q, respectively). If these variations

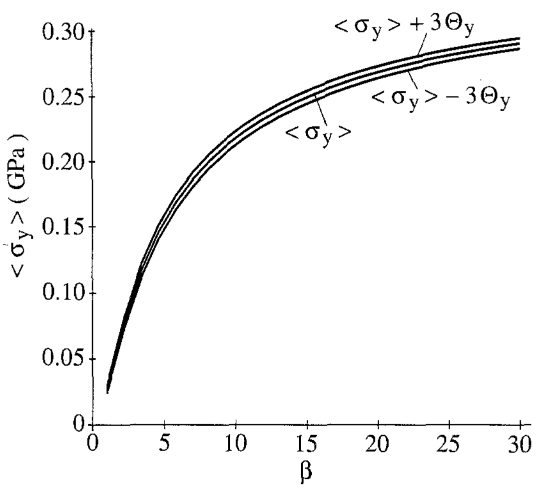

Figure 5 The predicted yarn strength $\langle \sigma_y \rangle$ and its $\pm 3\Theta_y$ boundary.

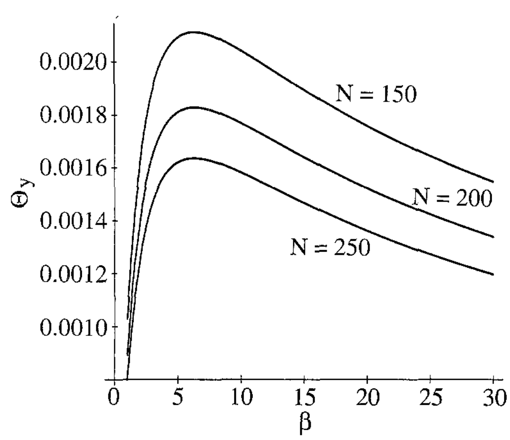

Figure 6 The standard deviation of the yarn strength Θ_y and the yarn-size effect.

are known, they can be superimposed directly on to the curves to give a more realistic prediction.

Fig. 6 shows the standard deviation of the yarn strength, Θ_y, plotted against the shape parameter at three different levels of fibre number, N. It can be seen from Equation 14 that a larger yarn size (a larger N value) will lower the variation. There is also a critical value (below 10 as seen in Fig. 6). When the shape parameter β increases to this critical value, the standard deviation values will increase to maximum. Once β is beyond the critical value, a larger β value, meaning more uniform fibres, will lead to a smaller variation in yarn strength.

10. Conclusion

Twisted fibre structures have a unique strength-generating mechanism, in which the force that is breaking the structure is, at the same time, strengthening it. This study stresses the fact that fibre strength is not strictly an intrinsic property but rather dependent on the structure and loading environment it is utilized for. Because of the lateral constraint, fibres in a yarn are stretched segment by segment as indicated by the fragmentation process. Because the lateral pressure increases together with the increasing yarn strain, the length of each fibre segment is hence decreasing to a much shorter value than the original fibre length. This will increase the strength of the fibre segment as well as the ultimate yarn strength, due to the fibre strength and length dependence. There is little difference between filament yarn and staple yarn at high twist level for strength prediction purposes. They can be treated exactly the same using the method proposed in this paper. Yarn strength can be more accurately predicted using the present theory if the variations in yarn fibre-volume fraction and in yarn twist value are provided.

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